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The effect of surface tension variation on filmwise condensation and heat transfer on a cylinder in cross flow

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Abstract—The earlier analyses of Nusselt and Rose which predict heat transfer in a condensing cross flow over an infinite cylinder are extended to account for the Marangoni effect. The influence on local heat transfer is notable for cases of high vapor velocity. The average values of heat transfer are not as easily compared with experimental results due to a singularity in the solution corresponding to the rapid thickening of the condensate layer.

INTRODUCTION

In 1871 Dr Carl Marangoni [1] published his observations of an oil drop spreading on a water surface. Marangoni made his observations at "the great basin in the Tuilerien in Paris" by throwing an oil-soaked sponge into the water. He explained what he observed by arguing that the flow was surface tension driven. Surface tension as a driving force has come to be generally referred to as Marangoni effect [2]. Marangoni effect has been reported to significantly impact mass transfer in falling films where a surfactant is present [3–5]. In this study, a mechanism whereby Marangoni effect may be manifested during the condensation of a flowing pure vapor is considered to determine if such phenomena may influence the heat transfer and condensate layer development.

Laminar filmwise condensation from a saturated vapor onto an isothermal circular cylinder in cross flow has received considerable attention, as evidenced in the reviews of Lee and Rose [6] and Westwater [7]. However, there are many important cases remaining to be studied, and research in this area is still ardent. Some recent progress has been reviewed by Eckert *et al.* [8].

For the condensation of water there are situations where the analysis of Nusselt [9] is satisfactory. However, there are many circumstances where Nusselt theory underpredicts the heat transfer and much effort has been directed at studying the impact of Nusselt's assumptions under such conditions. Many investigators have employed a boundary-layer formulation, first suggested by Sparrow and Gregg [10], for analyzing laminar film condensation. However, few have considered surface tension effects on the condensate layer in their analyses.

Rose [11] performed an analysis with the basic assumptions of Nusselt, but included vapor shear and the pressure gradient due to flow. Rose neglected surface tension effects in his analysis, and his results still tend to underpredict heat transfer at high vapor velocities.

Some researchers have studied the impact of surface tension on the curved condensate surface, primarily with respect to leading edge effects [12–14]. Krupiczka [15] used the variation in curvature of the condensate film, and neglecting all other effects, accounted for a pressure gradient term due to surface tension. However, these studies did not specifically attempt to extend the Nusselt, or the Rose analyses accounting for surface tension variation. Such an extension will be made in this analysis.

The purpose of this study is to simultaneously account for the pressure gradient, vapor shear and surface tension effects on the condensate layer. The variation of the surface tension due to a varying saturation state, specifically, is to be reckoned with in this study, as it has never before been examined in this context.

ANALYSIS

The physical model is presented in Figs. 1 and 2. The cylinder causes a pressure gradient in the flowing vapor which in turn leads to a streamwise pressure gradient in the condensate film. Since the liquid/vapor interface is at saturation everywhere, the saturation temperature at the liquid/vapor interface must vary with the pressure around the circumference of the cylinder. The surface tension of water is a strong function of temperature. Thus, a surface tension induced stress is present at the interface, as well as a vapor drag induced stress. The variation of the saturation state around the circumference of the cylinder is the basis of the extension of the Nusselt and Rose analyses.

The following assumptions are fundamental to the analysis.

NOMENCLATURE

c_p	specific heat at constant pressure
D	cylinder diameter
Fr	Froude number, $U_\infty/(gD)^{1/2}$
g	acceleration due to gravity
h	convective transfer coefficient
Ja	Jacob number, $c_p\Delta T/\lambda$
K	thermal conductivity
$\dot{m}''$	mass flux (per unit area)
Nu	Nusselt number, hD/K
P	pressure
Pr	Prandtl number, $c_p\mu/K$
$\dot{q}''$	heat transfer rate due to conduction, per unit area
$\widetilde{Re}$	Reynolds number, $\rho UD/\mu$ (two phase)
T	temperature
u	streamwise velocity in the condensate film
U	streamwise velocity, in the vapor region
x	longitudinal coordinate, measured from the upstream stagnation point
y	radial coordinate, measured from the surface of the cylinder.

Greek symbols

δ	thickness of the condensate layer
λ	latent heat of vaporization
μ	dynamic viscosity
ξ	$\partial\sigma/\partial T_{sat}\cdot \mathrm{d}T_{sat}/\mathrm{d}P$ (equation (7b))
ρ	density
σ	surface tension
τ	shear stress
ϕ	angle, measured from the upstream stagnation point [radians].

Subscripts

lv	liquid/vapor interface
sat	saturation state
v	vapor phase
w	cylinder surface
0	stagnation point
∞	undisturbed free stream, upstream of the cylinder.

(1) Condensation takes place from the flowing saturated pure vapor onto the isothermal right circular cylinder of infinite length.

(2) The condensate film is laminar, has constant density, and its inertial forces are negligible when compared with the gravitational and viscous forces. The condensate layer only has a velocity component in the x-direction.

(3) The external vapor flow can be approximated by the potential flow solution (over the base cylinder).

(4) Heat transfer through the condensate layer is by conduction.

(5) The temperature is continuous across the liquid/vapor interface, i.e. the so-called interfacial resistance is zero.

(6) Thermophysical property variation in the radial direction is negligible.

(7) The ratio of condensate thickness to cylinder radius is small, i.e. $2\delta/D \ll 1$.

Further assumptions will be introduced in the development, as required. The conservation of mass, momentum, and energy may then become

$$\dot{m}'' = \frac{2\rho}{D}\frac{\mathrm{d}}{\mathrm{d}\phi}\left[\int_0^\delta u\,\mathrm{d}y\right] \quad (1)$$

$$\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \rho g\sin\phi - \frac{2}{D}\frac{\partial P}{\partial \phi} = 0 \quad (2a)$$

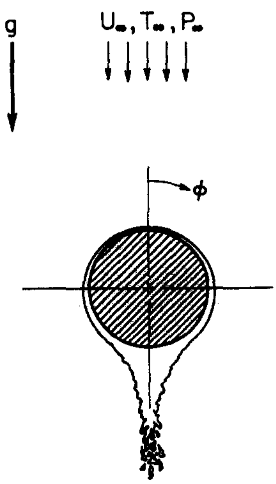

FIG. 1. Physical model.

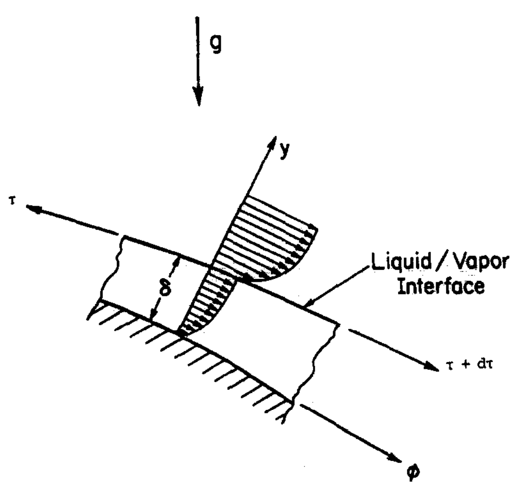

FIG. 2. Condensate layer.

$$\rho g \cos\phi - \frac{\partial P}{\partial y} = 0 \tag{2b}$$

$$\dot{m}'' = \frac{K\Delta T}{\delta\lambda} \tag{3}$$

where

$$\Delta T = \Delta T(\phi) = T_{sat} - T_w$$

and

$$P = P(\phi) = \tfrac{1}{2}\rho_v(u_\infty^2 - u_{lv}^2) + P_\infty$$

subject to the conditions

$$u = 0 \quad @ \; y = 0 \tag{4a}$$

$$\mu\frac{\partial u}{\partial y} = \tau_{lv} \quad @ \; y = \delta. \tag{4b}$$

The momentum equation may be integrated to yield the velocity distribution in the condensate film (also see ref. [11])

$$u(y) = \frac{1}{\mu}\left[\tau_{lv} y - \left(\rho g \sin\phi - \frac{2}{D}\frac{dP}{d\phi}\right)\left(\frac{y^2}{2} - \delta y\right)\right]. \tag{5}$$

The shear at the interface is assumed to be due to vapor shear, and Marangoni effect. The vapor shear is determined from a simple momentum balance in the vapor boundary-layer flow region

$$\tau_{lv} \cong -\frac{\partial}{\partial x}\int_{D/2}^{\infty} U^2(y,\phi)\rho_v \, dy$$

where

$$\dot{m} \cong -\frac{\partial}{\partial x}\int_{D/2}^{\infty} \rho_v U(y,\phi)\, dy.$$

This in turn can be approximated by an expression due to Shekriladze and Gomelauri [16]

$$\tau_{lv\,(\text{vapour shear})} \cong \dot{m}'' U_{lv}. \tag{6a}$$

The Marangoni stress at the liquid/vapor interface must balance the surface tension forces, hence

$$\tau_{lv\,(\text{Marangoni})} = \frac{\partial\sigma}{\partial T_{sat}}\frac{2}{D}\frac{\partial T_{sat}}{\partial\phi}. \tag{6b}$$

By making use of the chain rule, the expression for Marangoni shear can be written in a more convenient form

$$\tau_{lv\,(\text{Marangoni})} = \xi\frac{2}{D}\frac{\partial P}{\partial\phi} \tag{7a}$$

where

$$\xi = \frac{\partial\sigma}{\partial T_{sat}}\frac{\partial T_{sat}}{\partial P} \tag{7b}$$

and with the assumptions used, P is only a function of ϕ, and T can similarly be expressed as only a function of P.

The quantity ξ is an assayable physical property. The definition of ξ suggests the Clausius–Clapeyron equation may be useful in its estimation. The behaviors of the surface tension and ξ for water are given in Figs. 3 and 4. Under any circumstances, this property can be easily evaluated with a computer program.

The flow outside the vapor boundary layer is given by the potential flow solution. At the interface (for $\delta/D \ll 1$) it follows that

$$U_{lv} = 2U_\infty \sin\phi \tag{8a}$$

and

$$\frac{\partial P}{\partial\phi} = -4\rho_v U_\infty^2 \sin\phi\cos\phi. \tag{8b}$$

Using equations (6)–(8) and the dimensionless groupings, the velocity distribution, equation (5), becomes

$$\frac{U}{U_\infty} = \frac{2\delta}{D}\frac{y}{\delta}\widetilde{Re}\sin\phi\left\{\frac{\dot{m}''}{\rho U_\infty} + \frac{4\delta\rho_v}{D\rho}\cos\phi\left[1 - \frac{y}{2\delta} - \frac{\xi}{\delta}\right] + \frac{1}{2Fr^2}\frac{\delta}{D}\left[1 - \frac{y}{2\delta}\right]\right\}. \tag{9}$$

Equation (9) can be integrated to give

$$\int_0^\delta u\, dy = 2DU_\infty\frac{\delta^2}{D^2}\widetilde{Re}\sin\phi\left\{\frac{\dot{m}''}{2\rho U_\infty} + 2\frac{\rho_v}{\rho}\frac{\delta}{D}\cos\phi \times\left[\frac{2}{3} - \frac{\xi}{\delta}\right] + \frac{\delta}{6Fr^2 D}\right\}. \tag{10}$$

Together with equations (1) and (3) the above leads to (in terms of dimensionless groupings)

$$\frac{Ja}{Pr}\frac{D}{\delta} = 4\widetilde{Re}\frac{\partial}{\partial\phi}\left\{\widetilde{Re}\frac{\delta^2}{D^2}\sin\phi\left[\frac{Ja\,D}{2Pr\,\widetilde{Re}\,\delta} + 2\frac{\rho_v}{\rho}\cos\phi \times\left(\frac{2\delta}{3D} - \frac{\xi}{D}\right) + \frac{\delta}{6Fr^2 D}\right]\right\}. \tag{11a}$$

The Reynolds number can be included with δ by defining

$$\delta^* = \frac{\delta}{D}\widetilde{Re}^{1/2}.$$

Furthermore, for convenience let

$$\xi^* = \frac{\xi}{D}\widetilde{Re}^{1/2}.$$

Equation (11a) then becomes

$$\frac{1}{\delta^*} = 4\frac{\widetilde{Re}^{1/2}Pr}{Ja}\frac{d}{d\phi}\left\{\frac{\delta^{*2}\sin\phi}{\widetilde{Re}^{1/2}}\left[\frac{Ja}{2Pr\,\delta^*} + 4\frac{\rho_v}{\rho}\cos\phi \times\left(\frac{\delta^*}{3} - \frac{\xi^*}{2}\right) + \frac{\delta^*}{6Fr^2}\right]\right\}. \tag{11b}$$

The first term on the right-hand side is due to vapor shear, the third due to gravity while the second includes pressure gradient and Marangoni effects.

With the property dependence assumptions equation (11b) can be rearranged to facilitate the solution of $\delta^* = \delta^*(\phi)$ into

$$\frac{d\delta^*}{d\phi} = \frac{\frac{1}{2} - \frac{\rho_v Pr}{\rho Ja}(\frac{8}{3}\delta^{*4} - 4\delta^{*3}\xi^*) + 4\frac{\rho_v Pr}{\rho Ja}\delta^{*2}\sin\phi\left[(\xi^{*\prime} + \frac{4}{3}\delta^* - 2\xi^*)\delta^*\cos\phi - \frac{Ja'\rho}{Pr\,\rho_v}\right] - \delta^{*2}\cos\phi\left(1 + \frac{Pr\,\delta^{*2}}{3Ja\,Fr^2}\right)}{\delta^*\sin\phi\left[1 + \frac{Pr}{Ja\,Fr^2}\delta^{*2} + 8\frac{\rho_v Pr}{\rho Ja}\cos\phi(\delta^{*2} - \xi^*\delta^*)\right]} \quad (12)$$

Symmetry requires that at the upstream stagnation point

$$\left.\frac{d\delta^*}{d\phi}\right|_{\phi=0} = 0. \quad (13)$$

This then implies, if δ^* at $\phi = 0$ is given by δ_0^*

$$\tfrac{2}{3}\delta_0^{*4}\left(8\frac{\rho_v Pr}{\rho Ja} + \frac{Pr}{Ja\,Fr^2}\right) - 8\frac{\rho_v Pr}{\rho Ja}\xi_0^*\delta_0^{*3} + 2\delta_0^{*2} - 1 = 0. \quad (14)$$

Two dimensionless groups dominate the flow behavior

$$\frac{\rho_v}{\rho}\frac{Pr}{Ja} = \Pi_1 \quad (15a)$$

and

$$\frac{Pr}{Ja\,Fr^2} = \Pi_2 \quad (15b)$$

for compactness then equations (9), (12) and (14) can be rewritten as

$$\frac{U}{U_\infty} = 2\delta^*\sin\phi\left\{\frac{y^*}{\delta^{*2}} + 4\Pi_1 y^*\cos\phi\left(1 - \frac{y^*}{2\delta^*} - \frac{\xi^*}{\delta^*}\right) + \Pi_2\frac{y^*}{2}\left(1 - \frac{y^*}{2\delta^*}\right)\right\} \quad (16)$$

$$\frac{d\delta^*}{d\phi} = \frac{\frac{1}{2} - \Pi_1(\frac{8}{3}\delta^{*4} - 4\delta^{*3}\xi^*) + 4\Pi_1\delta^{*2}\sin\phi\left[(\xi^{*\prime} + \frac{4}{3}\delta^* - 2\xi^*)\delta^*\cos\phi - \frac{Ja'}{\Pi_1 Ja}\right] - \delta^{*2}\cos\phi\left(1 + \frac{\Pi_2}{3}\delta^{*2}\right)}{\delta^*\sin\phi[1 + \Pi_2\delta^{*2} + 8\Pi_1\cos\phi(\delta^{*2} - \delta^*\xi^*)]} \quad (17)$$

and

$$\tfrac{2}{3}\delta_0^{*4}(8\Pi_1 + \Pi_2) - 8\Pi_1\xi_0^*\delta_0^{*3} + 2\delta_0^{*2} - 1 = 0 \quad (18)$$

where

$$y^* = \frac{y}{D}Re^{1/2}.$$

An infinite growth rate will occur when

$$1 + \Pi_2\delta_{cr}^{*2} + 8\Pi_1\cos\phi_{cr}(\delta_{cr}^{*2} - \delta_{cr}^*\xi_{cr}^*) = 0 \quad (19)$$

whereas separation in the condensate layer is given by the condition that the wall shear stresses are zero

$$1 + 4\Pi_1\cos\phi_s(\delta^{*2} - \delta^*\xi^*) + \frac{\Pi_2}{2}\delta^{*2} = 0. \quad (20)$$

Equations (19) and (20) give Rose's solution when $\xi^* = 0$. Rose [11] suggests that the finite growth rate conditions are usually more restrictive and $\phi_{cr} < \phi_s$. The analysis presented is valid only up to the point where separation of the liquid or vapor layer occurs, or where an infinite growth rate occurs.

The heat flux through the condensate layer is given by

$$\dot{q}'' = \frac{K\Delta T}{\delta} = \frac{K\Delta T}{D}\frac{\tilde{R}e^{1/2}}{\delta^*}. \quad (21)$$

In that case the average heat flux would given by

$$\dot{q}''_{ave} = \bar{h}\Delta T_\infty = \frac{1}{\pi}\int_0^{\phi_{cr}} \dot{q}''\,d\phi. \quad (22)$$

From equation (22), an average Nusselt number is obtained

$$\overline{Nu} = \frac{1}{\pi}\int_0^{\phi_{cr}} \frac{\tilde{R}e^{1/2}}{\delta^*}\frac{\Delta T}{\Delta T_\infty}\,d\phi \quad (23)$$

where

$$\Delta T_\infty = T_\infty - T_{surf}: \text{constant with } \phi$$

and

$$\Delta T = T_{sat} - T_{surf}: \text{dependent on } \phi.$$

A significant difference between these expressions and those of Rose [11] is the inclusion of the saturation temperature variation.

ORDER OF MAGNITUDE CONSIDERATIONS

Four effects have been identified in modeling condensate layer growth. These are vapor shear, gravitational, pressure gradient and surface tension. The classical analyses have considered the first two, Rose's extension added the third, while the present study includes the fourth effect and accounts for the variation of the saturation state due to the flow.

The analysis does permit an approximate analysis of the order of magnitude of these effects. By simply gathering the terms due to each effect in equation (17) the following compare the order of magnitude of the vapor shear gravity, and pressure gradient forces to those due to Marangoni effect:

$$\frac{\text{Marangoni}}{\text{vapor shear}} = r_v$$

$$= \frac{(1-2\sin\phi\cos\phi)(\delta^{*3}\xi^* + \frac{1}{2}\delta^{*3}\xi^{*\prime})}{\frac{Ja'}{4\Pi_1 Ja}\delta^{*2}\sin\phi + \frac{1}{4\Pi_1}\delta^{*2}\cos\phi} \quad (24)$$

$$\frac{\text{Marangoni}}{\text{gravity}} = r_g$$

$$= \frac{(1-2\sin\phi\cos\phi)(\delta^{*3}\xi^* + \frac{1}{2}\delta^{*3}\xi^{*\prime})}{\frac{\Pi_2}{12\Pi_1}\delta^{*4}\cos\phi} \quad (25)$$

$$\frac{\text{Marangoni}}{\text{pressure gradient}} = r_p = \frac{\left(\xi^* + \frac{\xi^{*\prime}}{2\delta^{*2}}\right)}{2/3}. \quad (26)$$

These may be simplified by making the assumption that all $\sin\phi$ and $\cos\phi$ terms are of the order of unity. Further, it will be assumed that δ^* is of the order of unity. This assumption is later verified for water, but may be checked for another fluid by using equation (14). Equations (24)–(26) then give

$$r_v = \frac{4\Pi_1\left(\xi^* + \frac{\xi^{*\prime}}{2}\right)}{\frac{Ja'}{Ja} + 1} \quad (27)$$

$$r_g = \frac{\xi^* + \frac{\xi^{*\prime}}{2}}{\frac{\Pi_1}{12\Pi_2}} \quad (28)$$

$$r_p = \tfrac{3}{2}\xi^* + \tfrac{3}{4}\xi^{*\prime}. \quad (29)$$

Interpretation of the dimensionless groups demonstrates that the Marangoni effect may manifest itself at high vapor velocities and/or low gravity.

METHOD OF SOLUTION

The condensate thickness is determined from equation (17); a non-linear ordinary differential equation. In the examples that follow the equation was solved numerically through a fourth-order Runge–Kutta integration. The step size taken was $\Delta\Phi = 1$ deg. Integrations with a step size twice as large produced a change in the predicted Nusselt number of less than 0.5%. For these calculations properties and their derivatives were calculated using a quadratic least squares curve fit to the thermophysical properties in the neighborhood of the saturation pressure at the current ϕ position of the integration.

The heat transfer was determined using equation (23) and the solution of equation (17) together with the potential solution for the pressure distribution and the property variation for the vapor. The integration, however, must be terminated when reaching the point of infinite condensate layer growth due to the singularity. Hence the total heat transfer is approximated by that up to the point of infinite condensate layer growth.

RESULTS

It was found that for water at low or moderate vapor flow rates the Marangoni effect was negligible. For example, at $\widetilde{Re} \leqslant 3.0(10^5)$, which is typical of the highest vapor velocities in the experimental studies of Lee [17], the predicted condensate layer thickness with Marangoni effect differed by less than 1% from that computed by neglecting the Marangoni effect. However, at high vapor velocities the effect was more pronounced. As a case to study we consider the experimental conditions given by Fujii *et al.* [18].

Under these high vapor velocity conditions, $\widetilde{Re} \approx 5.0(10^5)$, the predicted condensate layer thickness is influenced by the Marangoni effect. The predicted condensate layer behavior is shown in Fig. 5. The percent difference between the predictions with and without Marangoni effect is given in Fig. 6.

As can be observed from Fig. 6 the surface tension variation tends to thin the condensate layer near stagnation, and thicken it in the region of adverse pressure gradient. This is exactly as one would imagine. However, our intuition does not extend trivially to the heat transfer behavior. Accounting for the saturation state

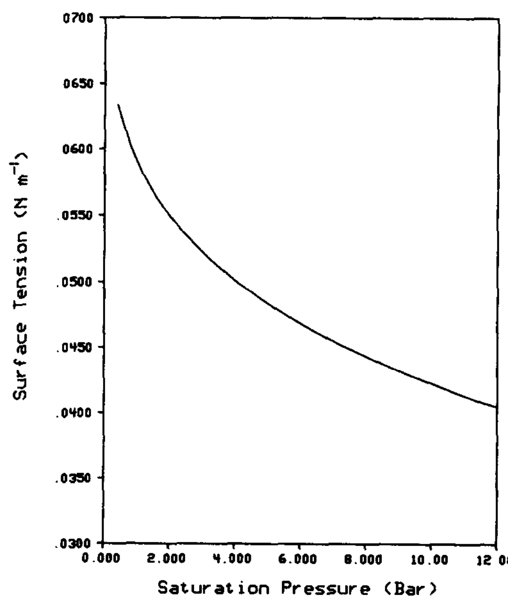

FIG. 3. Variation of the surface tension of water.

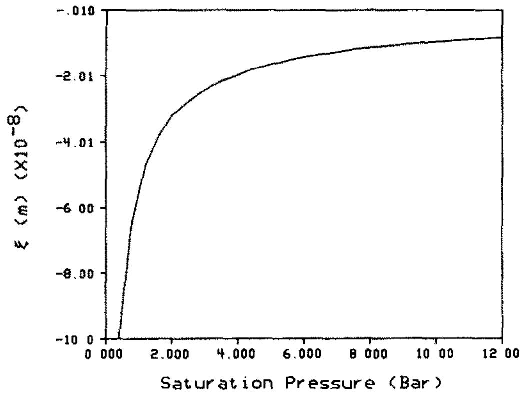

FIG. 4. Variation of ξ of water.

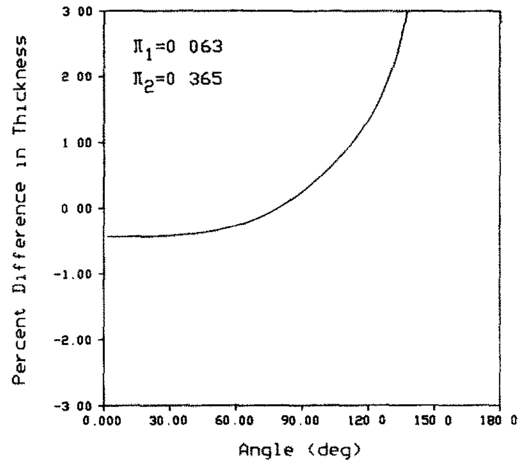

FIG. 6. Impact of Marangoni effect on condensate layer thickness.

variation alters the local heat transfer coefficients considerably. Since the temperature differences of equation (23) must remain within the integration we have the dual effects of the Marangoni shear, and the saturation temperature variation. The saturation temperature variation dominates the heat transfer at high vapor velocities. An illustration of the local Nusselt number variation with and without these effects is given in Fig. 7. Most notable is the more rapid decrease in local Nusselt number predicted in this study. The percent difference between these prediction with the inproved sets of equations agree with Fig. 6 it is apparent that this difference is driven by the saturation temperature variation.

The order of magnitude analysis suggested small Marangoni effects for regions such as those included in Lee's [17] data. Figure 9 confirms this. The prediction with the improved sets of equations agree with Rose's [11] prediction as well as the test data.

Figure 7 suggests that inclusion of the Marangoni effect (in cases of high vapor shear) while exhibiting a notable difference in local Nusselt number might not be as dramatic as in the average Nusselt number and could even lead to underprediction when compared to Rose's analysis. This is demonstrated in Fig. 10. The two predictions are now compared to Fujii *et al.*'s [18] data.

From these comparisons it appears that including the Marangoni effect makes for even more severe underprediction of the Nusselt number at high vapor velocities. However, it must be recalled that the analysis determined the average heat transfer integrating only up to the critical angle. The experiments, however, include the heat transfer taking place after the critical point. Had the experimental data reflected the heat transfer only up to the critical angle, the agreement between the predictions and the experiments would have been better. However, the effects beyond the critical angle are indeed small, due to the thickness of the condensate layer.

The apparent underprediction by the analysis may also be due to the influence of the vapor boundary layer on the condensate layer, which was not considered. At the high Reynolds numbers considered in this study interfacial waves may become important,

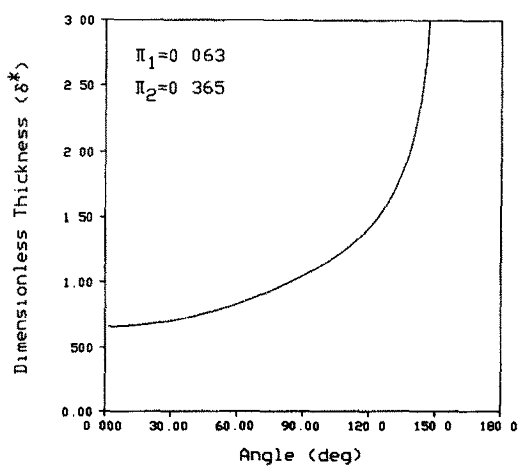

FIG. 5. Predicted condensate layer thickness.

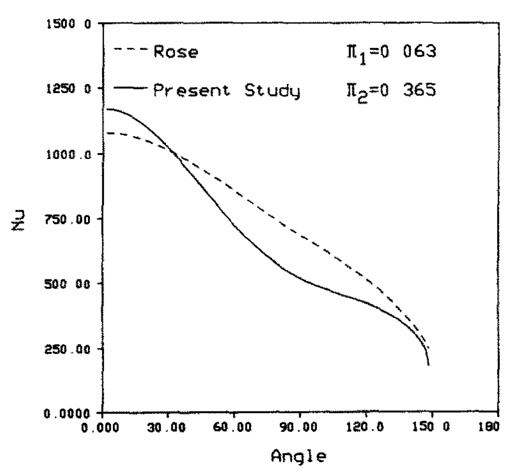

FIG. 7. Comparison of local Nu behavior.

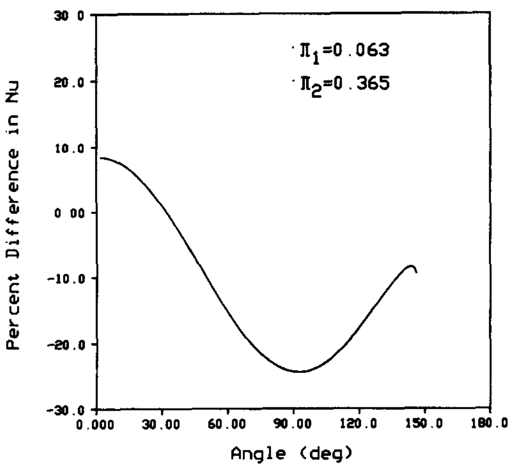

FIG. 8. Impact on local *Nu* behavior.

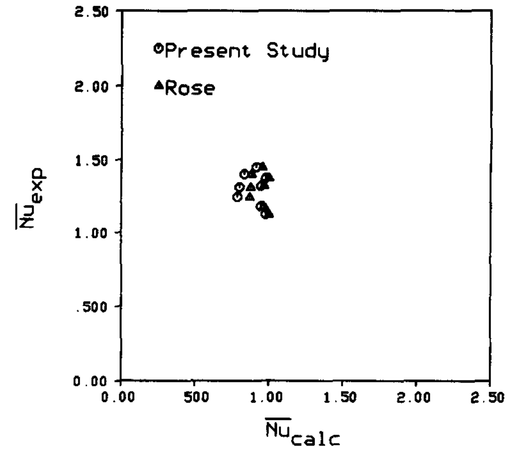

FIG. 10. Comparison to data of Fujii *et al.*

or turbulence of the liquid film may occur. These effects would serve to enhance the heat transfer, and may account for the experimental data being underpredicted. The enhancement due to waviness or rippling and turbulence has been reported by many investigators (see, for example, refs. [19–21]).

CONCLUSIONS

At high vapor velocities the condensate layer thickness can be influenced by the Marangoni effect, but this influence is not strong and for most realistic operating conditions can be expected to be less than 3%.

Local Nusselt numbers can be significantly influenced by the variation of the saturation state under high vapor velocity conditions, varying as much as 25% from previous predictions which neglect these effects. This observation may be meaningful with regards to the conjugate heat transfer problem encountered when considering tube temperature variation. Average Nusselt numbers are less sensitive to this since integrating around the circumference includes the increase near stagnation as well as the decrease near 90°. The net impact seems to be limited to about a 10% decrease in predicted average Nusselt numbers for high vapor velocity conditions (e.g. $\widetilde{Re} \geqslant 5(10^5)$).

The analysis has several areas where improvements may be made. Use of the potential flow solution may be replaced by a better description of the free stream velocity behavior. Questions regarding vapor boundary layer separation, interfacial waves, liquid film turbulence, and the influence of these on the heat transfer should be addressed. The physical significance of the critical angle, and its influence on heat transfer should be studied in greater detail.

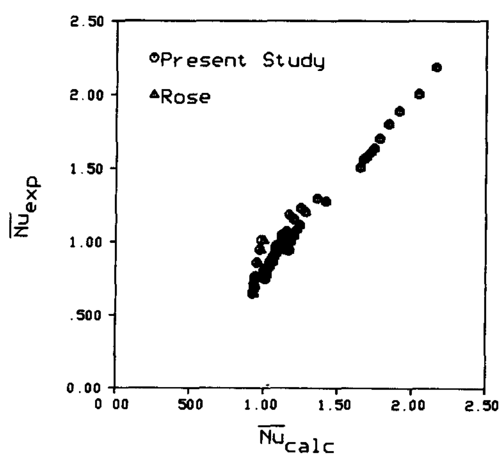

FIG. 9. Comparison to data of Lee.

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EFFET DE LA VARIATION DE TENSION INTERFACIALE SUR LA CONDENSATION EN FILM ET LE TRANSFERT DE CHALEUR SUR UN CYLINDRE EN ATTAQUE FRONTALE

Résumé—Les analyses antérieures de Nusselt et Rose qui prédisent le transfert thermique pour la condensation avec courant frontal sur un cylindre infini, sont élargies pour tenir compte de l'effet Marangoni. L'influence du transfert thermique local est importante dans le cas des grandes vitesses de vapeur. Les valeurs moyennes du transfert ne sont pas proches des résultats expérimentaux à cause d'un épaississement rapide de la couche de condensat.

EINFLUSS DER OBERFLÄCHENSPANNUNG AUF DEN WÄRMEÜBERGANG BEI FILMKONDENSATION AN EINEM QUERANGESTRÖMTEN ZYLINDER

Zusammenfassung—Die analytischen Untersuchungen von Nusselt und von Rose über den Wärmeübergang bei der Kondensation an einem unendlich langen querangeströmten Zylinder werden dahingehend erweitert, daß nun auch der Marangoni-Effekt berücksichtigt wird. Der Einfluß auf den örtlichen Wärmeübergang ist bei hoher Dampfgeschwindigkeit spürbar. Die Mittelwerte des berechneten Wärmeübergangs lassen sich nicht einfach mit Versuchsergebnissen vergleichen, da sich bei der Lösung infolge des schnellen Anwachsens der Filmdicke eine Singularität ergibt.

ВЛИЯНИЕ ИЗМЕНЕНИЯ ПОВЕРХНОСТНОГО НАТЯЖЕНИЯ НА ПЛЕНОЧНУЮ КОНДЕНСАЦИЮ И ТЕПЛОПЕРЕНОС НА ЦИЛИНДРЕ В ПОПЕРЕЧНОМ ТЕЧЕНИИ

Аннотация—Проведенные ранее Нуссельтом и Роузом исследования по теплопереносу в конденсирующемся поперечном течении над цилиндром бесконечной длины обобщаются с целью объяснения эффекта Марангони. Влияние поверхностного натяжения на локальный теплонеренос существенно при высоких скоростях пара. Вследствие сингулярности решения, соответствующего быстрому утолщению слоя конденсата, средние значения теплопереноса нельзя легко сопоставить с экспериментальными результатами.